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The USE of the
SECTOR,
IN THE
CONSTRUCTION
O F
SOLAR ECLIPSES.

WHEREIN,

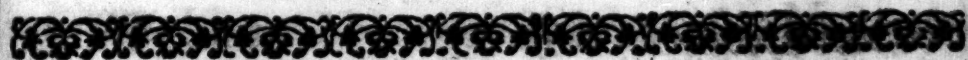
As a Proper EXAMPLE,

IS CONTAINED,

The Construction of the
GREAT ECLIPSE,

Which will happen *May 11. 1724.*

For *London, Edenburgh, Rome,*
and *Genoa.*



L O N D O N:

Printed for *Tho. Wright*, and sold by the
Bookfellers and Mathematical Instrument
Makers of *London and Westminster, 1721.*

T H E

P R E F A C E.

Courteous Reader,

I *Here present you with a Page or two, wherein is contained the Construction of the Appearances of Solar Eclipses for any given Latitude. I need not say any thing as to the Excellency of this Method beyond that of Calculation by means of Parallaxes, (which is extremely tedious and perplexed) that being too well known already, and especially by all those who have experienc'd the Difference.*

This Method was first publish'd by Mr. Flamsteed, and is contained in a Piece by him compiled, entitled, The Doctrine of the Sphere; which is annex'd to the First Volume of Sir Jonas Moore's System of Mathematicks. In the Preface to the said Doctrine of the Sphere, Mr. Flamsteed tells us, that he was not the first Inventor of this Method, but that Sir Christopher Wren had known the same Method several Years before him. And likewise,

The Preface.

While, discoursing with Mr. Halley, before he went to observe the Southern Constellations at St. Helena, the said Mr. Halley mentioned the Construction of Eclipses as possible, but did not communicate his Method.

It being then very probable, that each of the before-mentioned ingenious Persons might possibly have fallen seperately upon the same Method, I shall say no more as to the first Invention, but proceed to give some account of the Cause of my Publication of these few Pages.

It is obvious, that Sir Jonas Moore's System is none of the valuablest Pieces, (I mean as to the Matters contained therein) and that its Price is large, and not easie to be gotten, it being now a long time since it was printed. Also the First Part, of the Doctrine of the Sphere, in my Opinion, is of no great Value, or at least Use: For what is therein contained, is the Explication of the diurnal and annual Phenomena, according to the Copernican Hypothesis, which may be found in other Books enough of a much lesser Price, and by means of the Stereographical Projection of the Circles of the Earth upon the Plan of the Ecliptick, the Eye being in the South Pole, therein is shewn the Manner of solving Astronomical Problems: Such as finding the Sun's

The Preface.

right Ascension, Declination, &c. which is much more difficult to conceive, than by supposing the Earth's Stability, and the Sun's Motion.

Therefore all that we have valuable, is what is in the Second Part of the Doctrine of the Sphere; in which is not only contained this Method of Constructing Solar Eclipses, but likewise the Manner of Constructing of Stellar Eclipses; that is, the Moon's Appulses to fixed Stars. You are also taught in the said Second Part, how to find the Places of the Luminaries by means of Tables annex'd thereto, and then the Manner of Calculating the General Phases of Eclipses; as likewise the Reasons for the said Calculations.

But because the Tables are erroneous, (as Mr. Flamsteed himself owns) and will be quite thrown aside, when others made upon the Newtonian Theory are published, and the Manner of finding the Places of the Luminaries, and the Calculations of the General Phases of Eclipses are elsewhere shewn: Therefore the most curious Part of the said Doctrine of the Sphere, is the Construction of Solar and Stellar Eclipses.

I say for all the before-mentioned Reasons, and because some Persons, skilful enough in
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The Preface

Affirming, one that (possibly) know how to use
the Sector, which is absolutely necessary, in
the Construction of Solar Eclipses, and is,
that I have published these few Pages, in
which, I have the Vanity to believe, the Me-
thod is laid down something easier than Mr.
Flamsteed has done it.
I should have you observe, that a Sector
proper for this Work ought to be one of no less
Length than One Foot. For it is impossible
to do the Business tolerably exact with one of
a lesser Length; because the Lines of Sines
and Chords, on those that are lesser than One
Foot in Length, cannot well be divided into
smaller Parts than every 30 Minutes; whereas
those on a Foot Sector, are divided into every
15 Minutes, and on them you may reasonably
estimate every 5 Minutes; but if the Sector
were yet larger it would be better.
Moreover, Since the Construction of Stellar
Eclipses is very difficult and tedious, upon the
account of finding the true time of the Con-
junction of the Moon and a Star, and like-
wise it being a Curiosity not near so much ra-
ther than of a Solar Eclipse: (For its Use
in determining the Longitude at Sea is not so
great as some have imagined, the Moon's Ap-
pearances of fixed Stars being Appearances too sel-
dom happening, or if they happen'd oftner,
yet

The Preface.

yet would it be difficult enough to learn Seamen their Construction.) Therefore I shall be silent as to this Head, and content my self with what hereafter follows, hoping it will not be unacceptable.

Only note, That the Difference of Meridians of two Places whose Latitudes are known, may be also readily had from the Construction of Solar Eclipses, by observing the exact Times of the Beginnings or Endings of the same Eclipse at both the Places. For example; Suppose the Beginning of the Eclipse (in the Great Scheme,) at Rome, is 42 Minutes past 6; having set one Foot of your Compasses upon 42 Minutes past 6 in the Path of Rome, with the Extent of the Semi-diameter of the Penumbra make an Arc, cutting the Line of the Moon's Way; and the Point wherein it cuts it, will be about 52 Minutes past 5 at London. Whence the Difference of Meridians between London and Rome in time will be 50 Minutes; which converted into Degrees and Minutes, will be about 12 Degrees $37\frac{1}{2}$ and such is the Difference of Longitude between London and Rome.

For the Use of the Moon's Apogee, and Perigee, and the Moon's Nodes, and the Moon's Longitude at Sea is not so great as some have imagined, the Moon's Apogee to fixed Stars being observed too seldom, and the Moon's Nodes being observed too seldom, or if they happen'd often, it would be of little use.

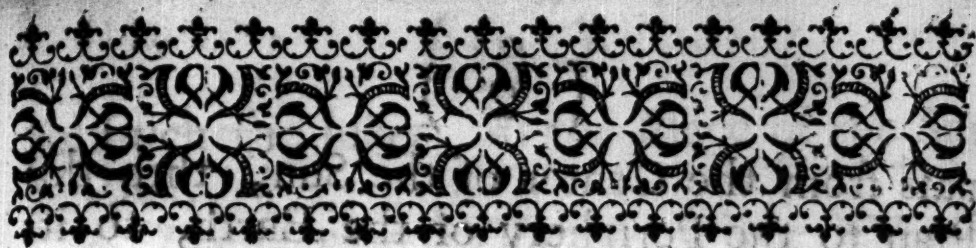
The Preface

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ans of two Places whose Latitudes are known,
may be also readily had from the Construction
of Solar Eclipses, by observing the exact Times
of the Beginnings or Endings of the same E-
clipse at both the Places. For example; Sup-
pose the Beginning of the Eclipse in the Great
Scheme, at Rome, is 44 Minutes past 6; the
beginning of the Eclipse at your Compasser upon a
Minutes past 6 in the Path of Rome, with
the Extent of the Semi-diameter of the Po-
lars make an Arc, cutting the Line of the
Moon's Way; and the Point wherein it cuts
it will be about 22 Minutes past 5 at Lon-
don. Whence the Difference of Meridians be-
tween London and Rome in time will be 20
Minutes; which converted into Degrees and
Minutes, will be about 3 Degrees 37; and
such is the Difference of Longitude between
London and Rome.

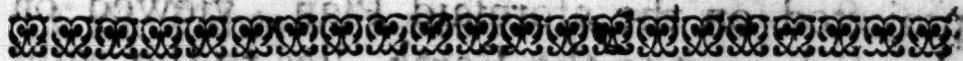
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THE
Use of the SECTOR
IN THE

Construction of Solar Eclipses.



DEFINITION I.

THE Path of a *Vertex*, is that Circle of the Earth which any Place or *Vertex* on its Superficies describes, in the Space of 24 Hours, by the Earth's Diurnal Revolution. Whence the Paths of *Vertices* are Circles parallel to the Equator.

DEFIN. II.

If a Plan be conceived to touch the Moon's Orbit in that Point, wherein a Line connecting the Centers of the Earth and Sun intersects the said Orbit, and stands at right Angles to the aforementioned Line: And if an infinite Number of right Lines be supposed to

B

pass

pass from the Center of the Sun, thro' this Plan to the Periphery of the Earth, to its Axis, as likewise to the Axis of the Ecliptick, and the Path of any *Vertex*; the said Lines will orthographically project, the Earth's Disk, its Axis, the Axis of the Ecliptick, and the Path of the *Vertex*, on the aforesaid Plan; and this is the Projection we are to delineate. This being presupposed, it will follow;

1. That when the Sun is in $\gamma \delta \pi \epsilon \eta \theta$, the Northern Half of the Earth's Axis projected on the aforesaid Plan, viewed on that Side next to the Earth, lies to the right Hand from the Axis of the Ecliptick: But if the Longitude of the Sun be in any of the Six opposite Sines, it lies to the left Hand from the Axis of the Ecliptick.

2. When the Sun's apparent Place happens to be either in $\gamma \delta \pi \epsilon \eta \theta$, the North Pole lies in the *illuminate* or *visible* part of the Disk; but otherways in the *obscure*.

3. When the Sun's Place in the Ecliptick is 90 Degrees distant from either Pole; that is, when the Sun is in the Equator, the Paths of the *Vertides*, or all Circles of the Earth parallel to the Equator, will be projected in right Lines upon the said Plan; but if the Sun's Place be lesser than 90 Degrees, the said Paths will be projected in Ellipses upon the said Plan, whose conjugate Diameters will be
fo

To much the lesser, as the Place of the Sun is lesser.

4. *The transverse Diameter of the Ellipse* representing any Path, is equal to double the right Sine of the Distance of the said *Vertex* from the *Pole*; that is, equal to twice the Co-Sine of the Latitude of the Place or *Vertex*: but the *Conjugate*, to the Difference of the right Sines of the Sum, and Difference of the Distances of the Path and Sun from the *Pole*; that is, equal to the Sine of the Complement of the Sun's Declination added to the Co-Latitude of the Place, less the right Sine of the Difference of the Complement of the Sun's Declination, and the Co-Latitude of the Place.

5. The transverse Diameter lies at right Angles to the Earth's Axis, and the conjugate, coincides therewith.

CHAP. I.

To represent in Plano, the Path of a *Vertex* in the Earth's Disk, whose Distance from the North Pole is $38^{\circ} 32'$, the Sun's Place being in $10^{\circ} 40' 30''$ of Π semblable to that which will be projected on a Plan, touching the Earth's Orbit in that Point, by strait Lines produced from the Sun to the Earth.

HAVING drawn the Semicircle *Fig. 1.*
 HER, let it represent the
 Northern Half of the Earth's illuminate
 Disk

Disk (because the Sun is in *Gemini*) projected upon the said Plan, $\odot$ it's Center, the Point therein opposite to the Sun, $H\odot R$ an Arc of the Ecliptick passing through it. Upon $\odot$ raise $\odot E$, perpendicular to the Ecliptick HR , and the Point E wherein it intersects the Limb of the Disk, will be the Pole of the Ecliptick, and $\odot E$ it's Axis.

Again; Make $\odot E$ equal to the Radius of a Line of Chords (by Chap. 3 of the Use of the Sector) from which taking the Chord of $23^{\circ} 29'$, (the constant Distance of the two Poles) set it off both ways from E to B and C , draw the Line BC , in which the Northern Pole of the World shall be found.

Make BA , equal to AC , the half of this Line, the Radius of a Line of Sines (by Chap. 3 of the Use of the Sector) and therein set off the Sine of the Sun's Distance from the Solstitial Colure $19^{\circ} 19' 30''$, from A to P , on the left hand of the Axis of the Ecliptick, (because the Sun is in *Gemini*) and draw the Line $\odot P$, which will be the Axis of the Earth, and P the Place of North Pole in the illuminate Hemisphere of the Disk.

Or the Angle $E\odot I$, which the Axis of the Earth and Ecliptick, make with each other, may be more accurately determined by Calculation: For,

As Radius, ——— $90^{\circ} 00' 00''$ $10,000000$
to the Sine of the
Sun's Distance from $19^{\circ} 19' 39''$ $9,519734$
the solstitial Colure
So is the Tangent of
the Sun's greatest $23^{\circ} 29' 00''$ $9,637916$
Declination to the
Tangent of the In- $8^{\circ} 10' 54''$ $9,157687$
clinat. of the Axis.

Count the said $8^{\circ} 10' 54''$ in the Limb of the Disk from E to I, on the left hand, and draw the Line $\odot I$, this shall be the Axis; and the Point P wherein it intersects the Line BC, the Place of the Pole in the illuminate Disk.

The next thing required will be the Sun's Distance from the Pole; or, the Complement of his Declination, which will be found $67^{\circ} 57' 48''$, this added to the Distance of the Vertex from the Pole $38^{\circ} 32'$, make $106^{\circ} 29' 48''$, and the same $38^{\circ} 32'$ taken from $67^{\circ} 51' 48''$, gives $29^{\circ} 25' 48''$, the Meridional Distance of the Sun from the *Vertex*.

Make $\odot E$ the Radius of the Disk, to be the Radius of a Line of Sines, from which take the Sine of $73^{\circ} 30' 12''$, (the Complement of $106^{\circ} 29' 48''$ to a Semi-circle) and set it off in the Axis from $\odot$ to 12, it there gives the Meridional Intersection of the Nocturnal Arc of the Path with the Axis.

Take

Take the Sine of $29^{\circ} 25' 48''$, from the same Line of Sines, and set it off the same way from $\odot$ to M, and it there gives the Intersection of the Diurnal Arc of the Path with the Meridian. Whence M 12 will be the conjugate Diameter of the Path, it being the Difference of the Sines of $70^{\circ} 30' 12''$, and $29^{\circ} 25' 48''$; that is, the Difference of the Sines of the Sum, and Difference of the Distances of the Path and Sun from the Pole, which will be the conjugate Diameter of any Path.

Bisect 12 M in C, and thro' it draw C 6 at right Angles to the Axis of the Globe; and then taking the Sine of $38^{\circ} 32'$, the Distance of the Pole from the *Vertex*, set it off from C both ways to 6 and 6; then the Line 66 will be the *Transverse-diameter* of the Path, and C 6 the *Semi-diameter*.

Making C 6 equal to the Radius of a Line of Sines, if from the same you take the right Sines of 15, 30, 45, 60, 75 Degrees and set them off severally both ways from C in the transverse Diameter, and from the Points so found erect Perpendiculars, a 11, a 1, a 10, a 2, &c. equal to the Cosines of the said Arc's, taken from a Line of Sines, whose Radius shall be C 12, equal to CM, you will have 24 Points given, through which the Ellipsis representing the Path shall pass, which shall also shew the Place of the Vertex at every Hour of the Day.

Day. In the same manner may the Parts of an Hour be pricked down in the Path, in laying off the Sine of the Degrees and Minutes corresponding thereto from C towards b, and then raising Perpendiculars from the Points so found in the Semi-transverse, and setting off from the said Semi-transverse each way upon the Perpendiculars, the Sines of the Complements of the Degrees and Minutes corresponding to the aforesaid Parts of an Hour. As for Example; To denote half an Hour past 11 and 12, take the Sine of 7 Degrees 30 Minutes, and lay it off on both Sides from C to b and b; then take the Co-Sine of 7 Degrees 30 Minutes, and having raised the Perpendiculars b₁, lay off the said Sine Complement from b to ₁, and you will have the Points in the Periphery of the Ellipsis, for half an Hour past 11, and half an Hour past 12; and in this manner may the Path be divided into Minutes, if the Ellipsis be large enough.

Take this for another Example; Suppose I would represent upon the Plane of the Earth's Disk, the Path of *Gibraltar*, whose Latitude is 35 Degrees 32 Minutes North, and the Sun's Place is in 15 Degrees 45 Minutes of *Leo*.

Having, as before, drawn the *Fig. 2.* Semicircle HER, for the Northern Half of the Earth's illuminate Disk, and drawn OE Perpendicular to RH, as also drawn

drawn the Line CB, which is always equal to twice the Chord of the Sun's greatest Declination, 23 Degrees 29 Minutes; you must next make AB equal to a Radius of a Line of Sines, and then lay off from A to P, on the right Hand of the Axis of the Ecliptick, (because the Sun is in *Leo*) the Sine of the Sun's Distance from the Solstitial Colure 45 Degrees 45 Minutes; or, the Angle EOP may be more nicely determined by Calculation, as was before directed, and then OPI, will be the Axis of the World.

Now the Sun's Distance from the Pole; or, the Complement of his Declination is 73 Degrees 51 Minutes, which being added to the Complement of the Latitude 54 Degrees 28 Minutes, the Sum will be 128 Degrees 19 Minutes, and this taken from 180 Degrees, the remainder will be 51 Degrees 41 Minutes; also if 54 Degrees 28 Minutes, be taken from 73 Degrees 51 Minutes, the Difference will be 19 Degrees 23 Minutes.

Then if you make the Semidiameter of the Disk, the Radius of a Line of Sines, and lay off from the Center O to 12, the Sine of 51 Degrees 41 Minutes, the Point 12 in the Axis, will be the Meridional Intersection of the Nocturnal Arc of the Path, with the Axis; and if again you lay off the Sine of 19 Degrees 23 Minutes from O to M, you will have the Meridional Intersection of the Diurnal Arc of the

the Path with the Axis, whence M_{12} will be the conjugate Diameter of the Elliptical Path.

And if you bisect M_{12} in C , and draw the Line $6C6$ at right Angles to the Axis OL , and then lay off the Sine Complement of the Latitude 54 Degrees 28 Minutes, from C to 6 on each Side the Axis, you will have the transverse Diameter of the Path, which may be drawn and divided, as before directed, for that of Figure 1.

Note. When the elevated Pole is in the obscure Hemisphere of the Earth, that the diurnal Arc, or illuminated Part of the Path, is in that Part of the Ellipsis that lies nearest to the said Pole, but otherways in the more remote; and where the Ellipsis cuts the Limb of the Disk, are the Points on it from which the Sun appears to rise, and set, &c. And because these Points are necessary to be found, when an Eclipse happens near Sun Rising, or Sun Setting, they may be determined in the following manner.

Lay off the Sun's Declination 22° , $2'$, upon the Limb of the Disk from R to N , as also the Complement of the Latitude of 38° , $32'$, from R to P ; then draw the Line ON , and from the Point P let fall upon the Diameter RH , the Perpendicular PQ , cutting the Line ON in L . This being done, take the Extent OL , between your Compasses, and lay it off upon the Axis OL from

from $\odot$ to K; then draw a Line both ways from the Point K, parallel to the transverse Axis C δ of the Path, and the said Line will cut the Limb of the Disk in the Points q p of the Sun's Rising and Setting.

Or the Arc Ip may be more accurately determined by Calculation; for in the Triangle $\odot QL$, right angl'd at Q, are given the Angle $QL\odot$, equal to the Sun's Distance from the Pole; and the Side $Q\odot$ equal to the Sine of the Latitude: To find the Side OL, which is equal to the Sine Complement of the Arc Ip; the Canon is as the Sine of the Sun's Distance from the Pole, is to Radius; so is the Sine of the Latitude, to the Sine Complement of the Arc Ip, or Iq.

CH A P. II.

HAVING in the foregoing Chapter shewn how to draw the Path of any Vertex upon the Earth's Disk, as likewise to divide it, the next things necessary to be given, in order to construct the Phases of a Solar Eclipse in any given Place on the Earth's Superficies, are;

I. The apparent Time of the nearest Approach of the Moon to the Center of the Disk; or the time of the Middle of the Eclipse.

II. The

II. The nearest Distance of the Moon's Center from the Center of the Disk in her Passage over it; which is equal to her Latitude at the time of the Conjunction.

III. The Semi-diameter of the Disk at the time of the Conjunction.

IV. The Moon's Semi-diameter at the same time.

V. The Sun's Semi-diameter.

VI. The Semi-diameter of the Penumbra.

VII. The Angle of the Moon's Way with the Ecliptick; which is equal to the Angle that the Perpendicular to the Moon's Way forms with the Axis of the Ecliptick; and if the Argument of Latitude be more than 9 Sines, or less than 3, the said Perpendicular lies to the left Hand; if more, to the right, from the Axis of the Ecliptick.

VIII. The Hourly Motion of the Moon from the Sun at the time of the Conjunction.

Note, The Semi-diameter of the Disk, is always equal to the Difference of the Sun and Moon's Horizontal Paralaxes.

All these for the Solar Eclipse of May 11. 1724. will be as follows.

The apparent Time of the nearest Approach of H. M. Afternoon. of the Moon to the Center of the Disk, will be, } 5. 11.

The nearest Distance of the Moon's Center from the Center of the Disk, } 30. 14

The Semi-diameter of the Disk, } 61. 38"

The Moon's Semi-diameter, } 16. 42"

The Sun's Semi-diameter, } 15. 53"

The Semi-diameter of the Penumbra, } 82. 35"

The Angle of the Moon's Way with the Ecliptick, } 5. 37

The Hourly Motion of the Moon from the Sun, } 35. 18"

These being found from Astronomical Tables and Calculations, I shall shew how to draw the Line of the Moon's Way, or Path of the Penumbra, upon the Plan of the Earth's Disk, as it falls at the time of the Conjunction of *May 11. 1724.* and the Manner of dividing the same, for *London, Genoa, and Rome.*

Having drawn the Semi-circle HER *Fig. 1.* of the Earth's Disk, and the Paths of *London, Genoa and Rome*, by the Directions of the last Chapter, the Sun's Place being $61^{\circ} 38' 45''$, and the Latitude of *London* $51^{\circ} 31'$, that

that of *Genoa* $44^{\circ} 27'$, and that of *Rome* $41^{\circ} 51'$; you must next draw the Perpendicular to the Moon's Way; which is done thus; Take the Semi-diameter $\odot H$ of the Disk between your Compasses, and open your Sector so, that the Distance from 60 to 60 of Chords be equal to that Extent; then taking $5^{\circ} 37'$ parallelwise from the Lines of Chords, (which is the Angle of the Moon's Way with the Ecliptick, or the Angle that a Perpendicular to her Way makes with the Axis $E\odot$, of the same Ecliptick) lay it off upon the Limb of the Disk from E to F , on the right Hand of the Axis of the Ecliptick, because the Argument of Latitude is more than three Sines, and the Line $\odot F$ being drawn, will be the Perpendicular to the Moon's Way at the time of the general Conjunction, *May 11, 1724.*

Again, Take the Semi-diameter of the Disk between your Compasses, and open the Sector so, that the Distance from $61\frac{3}{4}$, the Semi-diameter of the Disk, on each Line of Lines be equal to that Extent; then the Sector remaining thus opened, take between your Compasses the parallel Extent of $32\frac{1}{2}$, the nearest Approach of the Moon to the Center of the Disk, and lay it off from $\odot$ to M , upon the Perpendicular to the Moon's Way; then, if upon the Point M , a Perpendicular, as $M G$, be drawn both ways, this

will

will be the Line of the Moon's Way, or Path of the Penumbra.

Now to divide the said Path into its proper Hours, which let be for *London*. The middle of the General Eclipse, or the time when the Moon's Center will be at M, happens at 12 Minutes past 5 in the Afternoon: say, as 1 Hour or 60 Minutes to $35^{\circ} 18'$, the hourly Motion of the Moon from the Sun; so is 12 Minutes the time more than 5 in the Afternoon, to $7^{\circ} 3'$ the Motion from 5 a Clock to the middle.

Your Sector remaining opened to the last Angle it was set too, take the Extent from $7^{\circ} 3'$ to $7^{\circ} 3'$ on each Line of Lines, and setting one Foot of your Compasses upon M, with the other make a Point on the Moon's Way to the right Hand; and this shall be the Place of the Penumbra at 5 a Clock in the Afternoon at *London*; which therefore denote with the Number V.

The Hourly Motion of the Moon from the Sun is $35^{\circ} 18'$, therefore take the parallel Extent of $35^{\circ} 18'$ on the Lines of Lines, between your Compasses, and setting one Foot upon V, with the other make Points on each side V, these shall shew the Place of the Moon's Center at the Hours of IV and VI; and if from these Points you farther set off the said Extent in the said Line, you may thereby find the Place of the Moon's Center for every

every Hour, whilst the Penumbra shall touch the Disk; and if the Space between every Hour be divided into 60 equal Parts, you shall have the Place of the Moon's Center in the Line of her Way, to every single Minute of Time.

Or, you may take the Semi-diameter of the Disk between your Compasses, and make a Scale thereof, in dividing it by means of the Sector, in the following manner: Open the Sector so, that the Distance between $61\frac{3}{8}$, the Semi-diameter of the Disk, and $61\frac{3}{8}$ on the Lines of Lines, be equal to the Semi-diameter of the Disk: This Distance lay off from A to B; then your Sector remaining thus opened, take between your Compasses successively, the parallel Distances of each Division to $61\frac{3}{8}$, and lay them off from A towards B, every 5 of which Number, and your Scale will be divided into Minutes: And by the same Method you may divide each Minute into Parts, serving for Seconds, if your Scale be long enough. Now your Scale being divided, you may make use thereof, for drawing and dividing the Path of the Penumbra, without the Sector: For $32\frac{1}{2}$ of these Parts of the Scale, gives you the nearest Distance of the Moon's Center to the Center of the Disk: Also $7\frac{3}{8}$ Parts of the said Scale, will be the Distance of the Center of the Penumbra from the Point M. at V a Clock, and

and $35\frac{1}{2}$ of the Parts of the Scale, will be the Distance from Hour to Hour, on the Path of the Penumbra.

Now to fix Numbers upon the said Path of the Penumbra, representing the Hours when the Moon's Center will be at the said Hours, at *Rome* and *Genoa*, we must have the Difference of Longitude between *London* and the said two Places given; as also, whether they are to the *East* or *West* from *London*; the Difference of Longitude between *London* and *Rome*, is $12^{\circ} 37'$; and between *London* and *Genoa*, is $9^{\circ} 37'$, they being both to the *East* from *London*. Each of these being reduced to Time, the former will be 50 Minutes, and the latter 38 Minutes, wherefore 5 a Clock for *Rome* on the Moon's Way, must be at 10 past IV, for *London*; and 6 a Clock at 10 Minutes past V, &c. understand the same for other Hours and Minutes. I have noted the Hours for *Rome* under the Line of the Moon's Way, with *Roman* Characters. Again, 5 a Clock on the Moon's Way for *Genoa*, must be set at 22 Minutes past 5 for *London*; and 6 a Clock, at 22 Minutes past 6, &c. I have noted the Hours for *Genoa* with small Figures over the Line of the Moon's Way.

Note, 10 Minutes, and 22, are each of them the Complement of 50 Minutes, and 38 Minutes to 60 Minutes.

C H A P. III.

To determine the apparent Time of the beginning, or end of a Solar Eclipse, the Time when the Sun shall be eclipsed to any possible Number of Digits, the Inclination of the Cusps of the Eclipse, and the Time of the visible Conjunction of the Luminaries, in any given Latitude.

THE Paths of London, Rome and Genoa; as also the Path of the Penumbra being drawn and divided, as directed in the two last Chapters for the Great Eclipse of 1724, which will be a very proper Example for sufficiently explaining this Method, take between your Compasses the Semi-diameter of the Penumbra $32\frac{3}{8}$, from the Lines of Lines on the Sector, it being first opened to the Semi-diameter of the Disk $61\frac{3}{8}$; or you may take it from your Scale, which being done, carry one Foot of your Compasses along the Line of the Moon's Way, from the right hand to the left; wherein find such a Point, that if the said Foot be set, the other Foot shall cut the same Hour or Minute, in the Path of the Vertex of any given Place; then the Points in the Paths upon which either of the Feet of your Compasses stand, will shew the Time of the Beginning of the Eclipse at that Place.

For Example; If you carry the Semi-diameter of the Disk along the Line of the Moon's Way, you will find that one Foot of the Compasses being set at *a*, on the Moon's Way, which is 40 Minutes past 5 in the Afternoon for London, the other Foot will fall on the Point *b* on the Path of London, which is likewise 40 Minutes past 5 in the Afternoon; wherefore the Beginning of the Eclipse at London will be at 41 Minutes past 5 in the Afternoon.

Again; If you carry still on the Foot of your Compasses, they remaining yet opened, to the Semi-diameter of the Disk, and find another Point on the Moon's Way, whereon if you fix one Point of your Compasses, the other shall cut the Path of the Vertex at the same Hour or Minute, which this stands upon in the Line of the Moon's Way; the Points whereon your Compasses stand in either Path, shall shew the Minute the Eclipse ends.

For Example; One Foot of the Compasses being set at *g* in the Path of the Vertex, which is 129 Minutes past 7 in the Afternoon, the other Foot will fall upon the Line of the Moon's Way at the same Hour and Minute, viz. 129 Minutes past 7; therefore the Eclipse ends at London, 129 Minutes past 7; but take notice, that the Line of the Moon's Way should be continued further out beyond 2 a Clock; that so the Point of the Compasses may

may fall upon the proper Minute, to wit, 29.
 Moreover, If one side of a Square be applied to the Heliptick *H B*, and so moved backwards or forwards, until the other side of the said Square cuts the same Hour or Minute in the Path of the Vertex, and line of the Moons Way; this same Hour or Minute, will be the time of the visible Conjunction of the Luminaries at the given Place.

For Example; When the perpendicular side of the Square cuts the Path of the Moon's Way at *e*, which is 37 Minutes past 6, the said side will likewise cut the Path of the Vertex for London at *c*, which is 37 Minutes past 6; therefore the Time of the visible Conjunction of the Luminaries at London will be 37 Minutes after 6.

Draw the Line *ab*, as also the Line *ob*, this shall represent the vertical Circle, and the Angle *oba*, will be the Angle that the vertical Circle makes with the Line connecting the Centers of the Sun and Moon, at the Beginning of the Eclipse at London.

Draw the Line *gm*, to wit, joyn the Points in the Path of the Vertex, and the Path of the Moon's Way, which shews the End of the Eclipse at London, and the line *ag*, then the Angle *agm*, will be that which the vertical Circle forms with the Line joyning the Centers of the Luminaries.

Take the Semi-diameter of the Sun, viz. 15th between your Compasses, either from

your Sector, opened as before directed, to the Semi-diameter of the Disk, or from your Scale, and with that upon the Center *c* (to wit, the Minute in the Path of *London*, whereat the time of the visible Conjunction happens) describe a Circle; this Circle shall represent the Sun.

Again; Take the Moon's Semi-diameter from your Sector, (remaining opened as before) or your Scale upon the Center *c*; (to wit, the Minute in the Path of the Moon's Way, whereat the true Conjunction happens at *London*) describe another Circle. This shall cut off from the former Circle, so much as the Sun will be eclipsed, at the Time of the visible Conjunction.

From *o* draw the Line *ocv*: This shall represent the vertical Circle, and *v* the vertical Point in the Sun's Limb, whereby the Position of the Cusps of the Eclipse, in respect of the Perpendicular passing thro' the Sun's Center, are plainly and easily had.

Produce *dc* till it intersect the Moon's Limb in *p*, then shall *pq* be the part of the Sun's Diameter eclipsed, at the time of the greatest Obscuration at *London*. And if the Sun's Diameter be divided into 12 equal Parts, or Digits, you will find *pq* to be 11 $\frac{1}{12}$ of those Parts or Digits.

Whence

Whence at *London*, $\S 05.41$

The beginning of the Eclipse, $\S 06.37$

May 11. 1724. at $\S 08.11^{66}$

The visible Conjunction of the $\S 07.20$

Luminaries, $\S 07.20$

The End, $\S 07.29$

After the same manner, the be-

ginning of the Eclipse at *Ge-* $\S 06.27$

nea will be,

Visible Conjunction, or middle

of the Eclipse, $\S 07.20$

The Sun will there Set eclips'd, and the

Eclipse will be Total.

And the beginning of the $\S 06.42$

Eclipse at *Rome* is,

The visible Conjunction, or middle, will there

be when the Sun is Set, and consequently

also the End.

I have, as you see in the Figure, also drawn

a fourth Path for *Edinburgh*, whose Latitude

is $55^{\circ} 56'$, and Longitude, about 3 Degrees

to the *West* from *London*. Wherefore for each

Hour in the Moon's Way for *London*, you

must account 12 Minutes more for the same

Hour at *Edinburgh*: That is, *For Example*,

5 a Clock on the Line of the Moon's Way from

Edinburgh, must stand at 12 Minutes past 5

at

at *London*. Understand the same for other
Hours, &c.

And by proceeding according to the Direc-
tions, before given, you will find,

At *Edenburgh*, H. M. Afternoon.

The beginning of the Eclipse at, 05. 22

The middle, 06. 20

The end, 07. 14

Note, The Path of the Moon's Way ought
to be continued out further to the left hand,
in order to determine the time of the End of
the Eclipse at *Edenburgh*.

If you have a mind to know at what time
any possible Number of Digits, or Minutes
shall be eclipsed at any Place in the Sun's an-
tecedent, or consequent Limb; divide the
Sun's Diameter into Digits, or Minutes; and
cut off the Parts required to be eclipsed from
the Semi-diameter of the Penumbra; then take
the remaining part of it between your Com-
passes, and carrying it along the Line of the
Moon's Way, find the first Point in it, in which
placing one Foot, the other will cut the same
Point in the Path of the Place that the fixed
Foot stands upon; then the Hour and Minute
in either Path upon which the Feet of your
Compasses stand, will be the Time of that
Obscuration.

As

As for Example; Suppose it was required to find at what time 6 Digits or $\frac{1}{2}$ of the Sun's Diameter shall be eclipsed in his antecedent Limb at *London*. Cut off $\frac{1}{2}$ of the Sun's Semi-diameter, from the Semi-diameter of the Penumbra, and carrying the Remainder, as directed, you will find, that if one Point of your Compasses be set at 6 Hours 9 Minutes in the Afternoon, on the Path of the Moon's Way, the other Point will also fall upon the same Hour and Minute in the Path of *London*; and therefore the Time when the Sun's antecedent Limb at *London* will be half eclipsed, will be at 9 Minutes past 6; and when its consequent Limb will be half eclipsed, will be at 5 Minutes past 7.

Now to determine the Position of *Fig. 2.* the Cusps of the the Eclipse; *For Example, at London:* Draw a Circle *ADBE*, representing the Sun's Body, and the right Line *ACB*, representing his vertical Diameter. This being done, lay off the Angle *oba* upon the Sun's Limb from *A* to *D*, draw the Diameter *ECD*, and the Point *D* will be the first Point of the Sun's Limb obscured by the Moon at the Beginning of the Eclipse.

Again; To determine the Position of *Fig. 3.* and Appearance of the Eclipse at the Time of the middle, or greatest Obscuration; take the Sun's Semi-diameter between your Compasses, and upon the Point *C* describe

Describe a Circle; then draw the vertical Diameter ACB , and make the Angle ACD equal to the Angle vcp , and draw the Diameter DCE . This being done, take the Moon's Semi-diameter between your Compasses; and having lay'd off from the Center C to E , the Distance ce in the first Figure; upon the Point E , as a Center, describe an Arc cutting the Sun's Limb and the Position, and Appearance of the Eclipse at the Time of the greatest Obscuration, or the Middle, at *London* will be as you see in the Figure,

Lastly; To determine the Position of the End of the Eclipse. Draw a Circle, (as in the 4th Figure) and cross it with the vertical Diameter ACB ; then make the Angle ACE equal to the Angle ogm , and draw the Diameter ED ; then will the Point E on the Limb of the Sun, be that which is last obscured, or whereat the Eclipse ends.

If you have a mind to find the Continuation of total Darkness at any Place where the Sun will be totally eclipsed, cut off the Semi-diameter of the Sun, from the Semi-diameter of the Penumbra, and taking the Remainder between your Compasses, carry it along the Line of the Moon's Way, and find the first Point in it, on which placing one Foot, the other will cut the same Hour in the Path of the Place; which Hour note down. Again, Carrying on further the same Extent of your Compasses, find

find two Points on the Paths of the Vertex and Moon's Way, which shall shew the same Hour and Minute on them both. This time also note down; then subtract the time before found from this time, and the Difference will be the time of Continuance of total Darknes.

CHAP. IV.

To determine the Time that the Penumbra is passing over the Disk; what Place of the Earth the Penumbra first touches; or in what Place the supreme Point of the Sun's vertical Diameter will at his Rising appear eclipsed: As also the manner of tracing the Passage of the Moon's Shadow over the Disk.

NOW to determine the time the Penumbra is passing over the Disk, you must consult *Fig. 5.* where EL is the Path of the Penumbra supposed to be drawn and divided, as has been already directed: Then if you describe with one Foot of your Compasses upon the Center S, with a Radius equal to the Sum of the Semidiameters of the Disk and Penumbra, the Arc's FF, FF, they will cut the Path of the Penumbra in the Points E and L, and then the Hours and Minutes of the said Path included between the said Points, will be the time that the Penumbra is passing over the Disk, which will be found 5 Hours

E

M

Minutes. Moreover, the Point L shows the time of the Beginning of the General Eclipse, and the Point E the End; the former will be at 1 Hour 41 Minutes, and the latter at 7 Hours 47 Minutes; these may also be found by Calculation, by means of the right lined right angled Triangles SME, SML, ES=SL, and SM being given.

Before we can find the next thing proposed, viz. the Place of the Earth the Penumbra first touches, we must find at what Place the Sun in a given time is vertical. Find the Sun's Place in the Ecliptick at the given time, and then its Declination will be the Latitude of the sought Place; and its Longitude from that Place from which the time is reckoned, will be had in converting the time from the Meridian, into Degrees and Minutes, of the Equinoctial: *As for Example;* The Longitude of a Place in whose Vertex the Sun is at 7 a Clock in the Morning at London, will be had by subtracting 7 from 12, and multiplying the Remainder 5 by 15, viz. 75° Degrees. Therefore that Place will be 75° Eastward from London. This being found, we proceed to determine the Place of the Earth, where the Penumbra first touches the Disk. Draw PC thro' P, the Pole of the Earth, where the Penumbra first touches the Disk, then in the right lined right angled Triangle SLM, you may find the Angle MSL,

MSL, and consequently the Angle BSL, because MSB is given. Again, In the right angled Spherical Triangle BPC, of the Superficies of the Earth, are given BP, the Sun's Declination, and the Arc BC, which is the Measure of the Angle BSL, where you may find the Arc PC, the Complement of the Latitude of the Place sought; as also the Angle BPC, whose Complement CPS to two right Angles, will be the Distance of the Meridians of the Place C, and that Place in which the Sun is vertical; and since this Place is known, the Place C will likewise be known.

Again; To find the Place of the Earth H, at which the Center of the Penumbra shall be at a given time, you will by the Hourly Motion of the Moon, have the Length MH; and then in the right angled Triangle SMH, the Side SH may be found, and the Angle HSM, (because MS is also given) and consequently the Angle HSB, since MSB is given. But SH is the Sine of the Arc of a vertical Circle passing thro' the Vertex of the Place H and the Sun, the Semi-diameter of the Disk being Radius; whence if you say, as the Semi-diameter of the Disk is to SH, so is a radius to another, you will have the Sine of an Arc, that will be the Sun's Distance from the Vertex H. Now in the oblique Spherical Triangle on the Superficies of the Earth SHH, are given SH, the Sun's Distance from the Pole of the World, the Angle HSH, and the Side MH, the Moon's Distance from the Vertex H. In this Triangle SHH, the Angle HSH is given, the Side MH is given, and the Side SH is given, so that the Angle HSH is given, the Side MH is given, and the Side SH is given, so that the Angle HSH is given, the Side MH is given, and the Side SH is given.

and SH the Sun's Distance from the Vertex, and
 to the Angle HSP . To find the Side HP , the
 Complement of the Latitude of the sought
 Place; and the Angle HPS , which is the
 Difference of Meridians of the Place H , and
 that Place whereat the Sun is vertical; but
 the Difference of the Meridians of that Place
 whereat the Sun is vertical, and that from
 which the time is computed, is given;
 whence the Place H will be known. And if,
 according to this Method, several Places on
 the Earth's Superficies are found, and they
 are joyned by Lines, you will have the Path
 of the Center of the Penumbra, over the Su-
 perficies of the Earth.

In like Manner; If you have a mind to de-
 termine the Breadth of the Moon's Shadow,
 raise a Perpendicular HG upon the Point H ,
 equal to the Difference of the Sun and Moon's
 apparent Semi-diameters; then in the right
 angled Triangle SQG , are given SQ and GQ ;
 whence GS may be found, and the Angle
 GSQ , and so the Angle GSB . But SG is the
 Sine of the Distance of the Place G , thro' which
 the Extremity of the Shadow passes; when
 the Center of it is at the Place H ; therefore
 the Latitude and Longitude of the Place G
 may be found by means of the spherical Tri-
 angle SPG , exactly in the same manner as be-
 fore, for the Latitude and Longitude of the
 Place H . And in the same manner may the
 Latitude

Latitude and Longitude of the Place R , be determined on the other Side the Path of the Penumbra: And if the Latitudes and Longitudes be thus found on both Sides the Path of the Penumbra, and they are joined by Lines, the said Lines will limit the Breadth of the Moon's Shadow, and all Places contained within those Limits, will be involved in total Darknes at the time of the Eclipse.

Note, After the same way may the Breadth of the Penumbra be determined.

The Cannon for finding the Side PC , of the right angled spherical Triangle PBC , is as Radius is to the Sine Complement of either of the given Sides PB , BC ; so is the Sine Complement of the other Side, to the Sine Complement of the sought Side PC .

And the Cannon for finding the Side PS of the oblique spherical Triangle PHS , by having the Sides PS , and HS , and the Angle PSH included between them given, is as the Cube of Radius is to the Rectangle under the Sines of the given Sides PH , SH ; so is the Square of the Sine of half the given Angle PSH , to half the Difference of the versed Sines of the sought Side PH , and the Arc of Difference between the given Sides, which half Difference doubled, and added to the versed Sine of the Difference of the two given

ven Sides, gives the veried Sine of the Side sought.

In the working this Case, it will be proper to use *Sherwin's Tables*, because there is required natural Sines, and veried Sines, standing against the artificial ones.

But this Case may likewise be solved at two Operations, by supposing a Perpendicular to be let fall, without the help of natural Tables: For,

First, If both the given Sides are equal, then it will hold, as Radius is to the Sine of either of the Sides, so is the Sine of half the given Angle, to the Sine of half the Side sought.

Secondly, If one of the given Sides be a Quadrant; this, by continuing out the other given Side to a Quadrant, will become a Case of right angled spherical Triangles; in which, besides the right Angle, instead of the quadrantal Side, there will be one Leg given, and its adjacent Angle, to find the other Angle, by the fourth Case of right angled spherical Triangles: Also, if the Angle given was 90 Degrees, it would be a Case of right angled spherical Triangles; in which, besides the right Angle, there would be both the Sides given, to find the Hypothenuse.

Thirdly,

Thirdly, If both the given Sides are lesser than Quadrants, it will hold, as Radius is to the Cosine of the given Angle; so is the Tangent of the lesser given Side, to the Tangent of a South Arc; then if the given Angle be lesser than 90 Degrees, subtract the said fourth Arc from the other given Side; but if it be more, from the other Sides Complement to 180 Degrees, and the Remainder is called the residual Arc; then say again, as the Cosine of the fourth Arc, to the Cosine of the residual Arc; so is the Cosine of the lesser given Side, to the Cosine of the Side sought.

Now the Side sought may be greater than a Quadrant, and so be doubtful: In order to solve which; when the given Sides are both either greater or lesser than Quadrants, and the Angle given acute, the Side sought will be lesser than a Quadrant.

But when the given Sides are one greater and the other lesser than a Quadrant, and the Angle comprehended between them is obtuse, the Side sought will be greater than a Quadrant.

F I N I S

Thirdly, If both the given Sides are lesser
than Quadrants, it will hold, as Radius is
to the Cosine of the given Angle; so is the
Tangent of the lesser given Side, to the Tan-
gent of a South Arc; then if the given Angle
be lesser than 90 Degrees, subtract the said
South Arc from the other given Side; but if
it be more, from the other Side, Comple-
ment to 180 Degrees, and the Remainder is
called the residual Arc; then say again, as
the Cosine of the South Arc, to the Cosine
of the residual Arc; so the Tangent of the
lesser given Side, to the Tangent of the Side
sought.



Now the Side sought may be greater than
a Quadrant, and so be doubtful: In order to
solve which; when the given Sides are both
either greater or lesser than Quadrants, and
the Angle given acute, the Side sought will
be lesser than a Quadrant.

But when the given Sides are one greater
and the other lesser than a Quadrant, and the
Angle comprehended between them is ob-
tuse, the Side sought will be greater than a
Quadrant.

Fourthly, If one of the given Sides be
equal to a Quadrant, and the other lesser
than a Quadrant, the Side sought will be
greater than a Quadrant.

And if both the given Sides be equal to
Quadrants, the Side sought will be equal to
a Quadrant.